

# How much is that black hole in the window?

(Robustness testing in Stellar Dynamical Modeling)

**Berkeley Big BH Bunch:**

**Emily Liepold**

**Jacob Pilawa**

**Matthew Quenneville**

**Chung-Pei Ma**

**Emily Liepold, postdoc @ UC Berkeley Astronomy**  
[emilyliepold@berkeley.edu](mailto:emilyliepold@berkeley.edu)

**The context of our science**

**Overview of how we do our science**

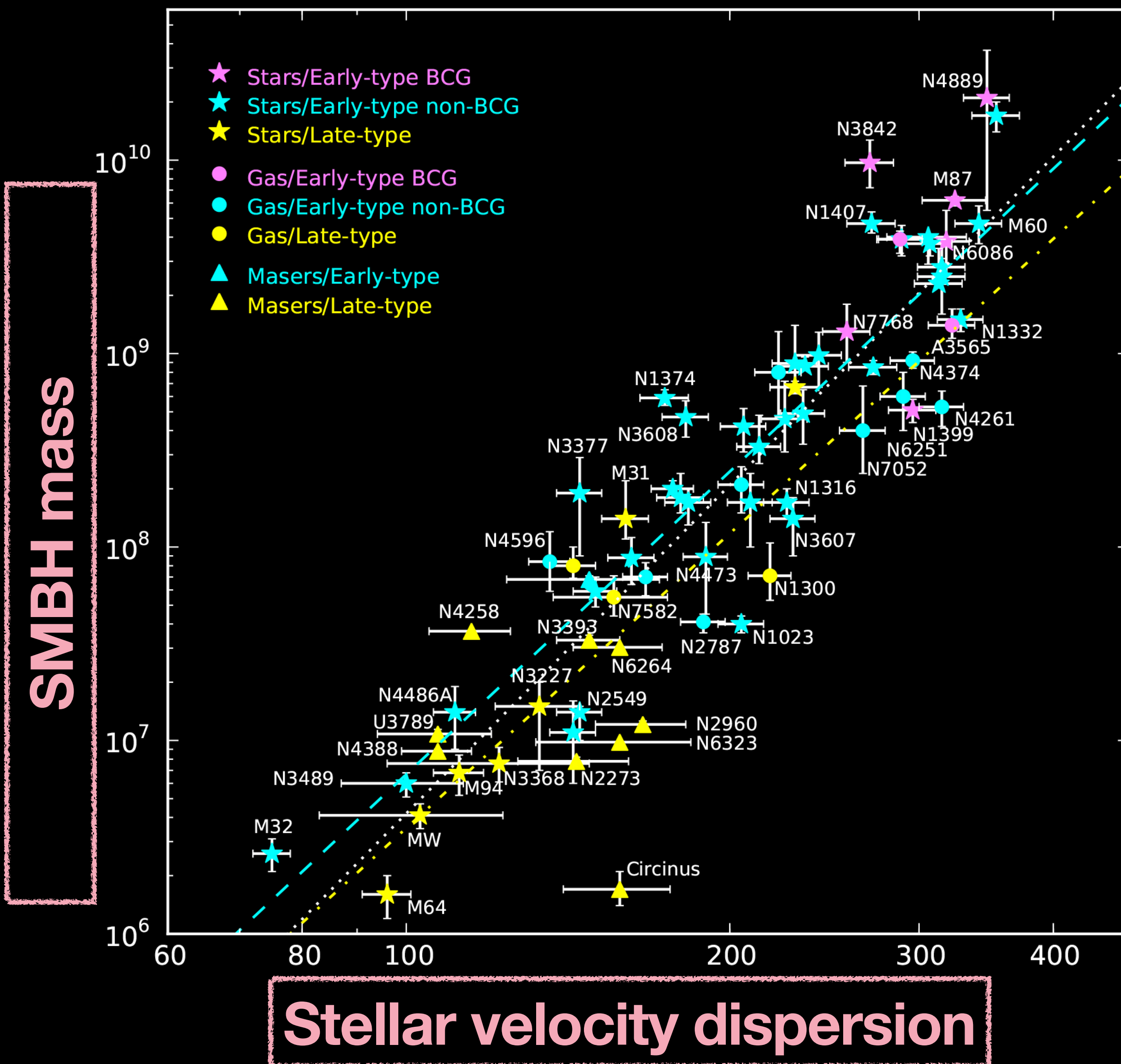
**The robustness problem in stellar  
dynamical modeling**

**(and our attempts to address that problem)**

**Case study: The Berkeley group's approach**

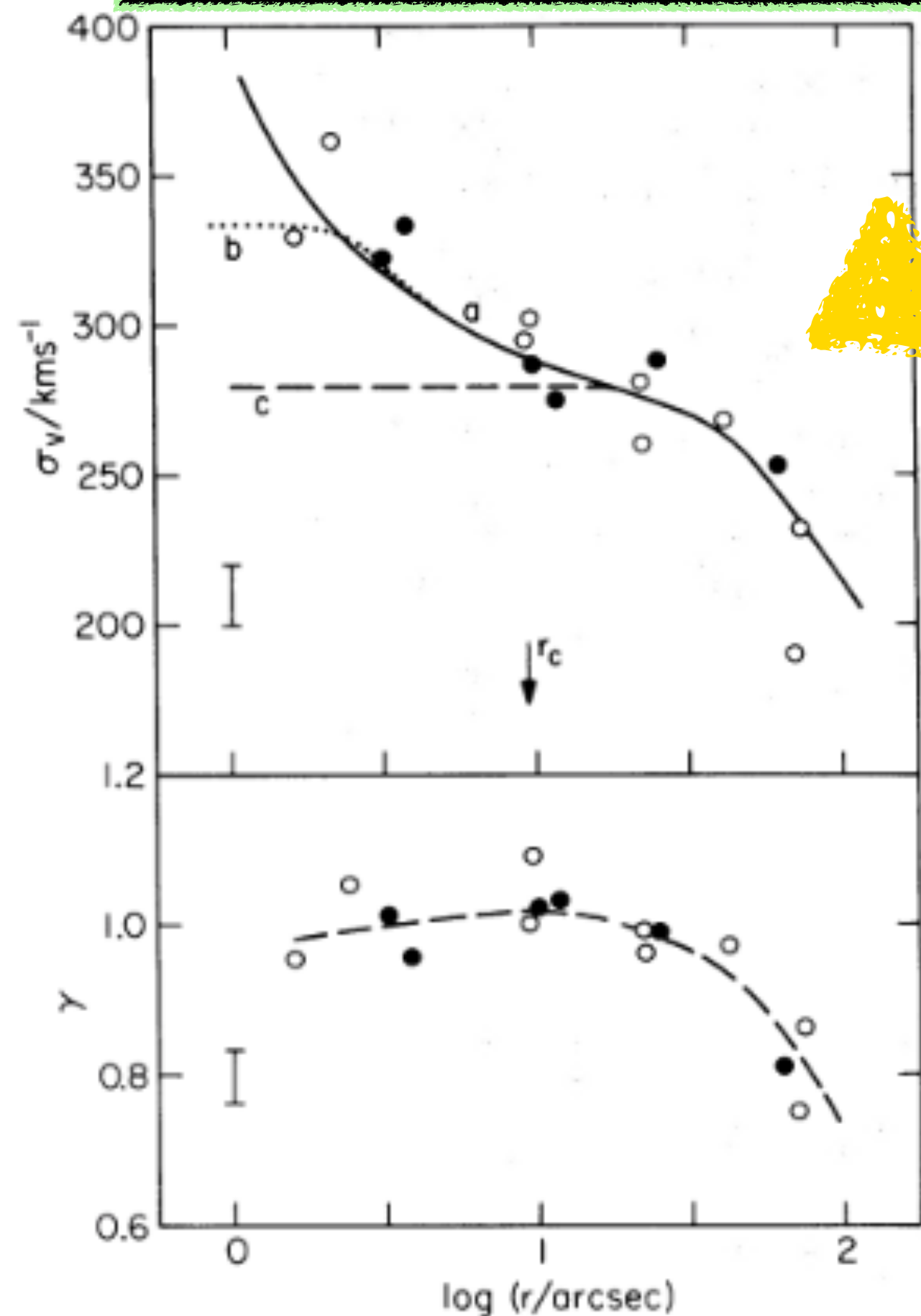
# SMBH masses are linked to galaxy properties

- Supermassive black hole masses are *correlated* with galaxy properties through coevolution
- The form of the correlation constrains models for coevolution
- Precise constraints on those correlations *require* precise measurements of individual SMBH masses and galaxy parameters



# High-precision stellar dynamical SMBH measurement is now possible

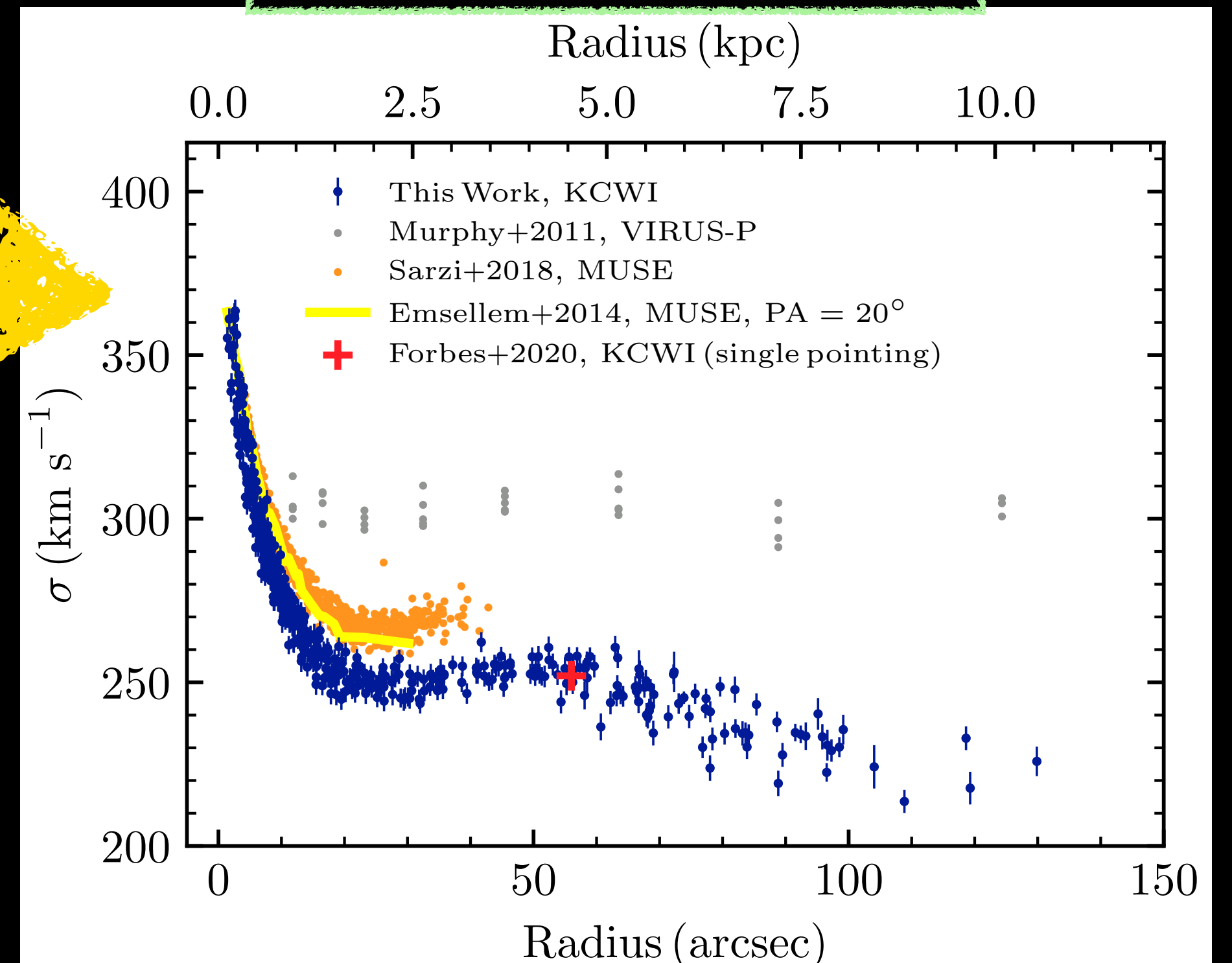
**Sargent+78**



**M87 velocity dispersion profile**

**Improved data  
+  
Improved models  
=  
Improved measurements**

**Liepold+23**



# Stellar dynamical modeling

Idea: stellar motions are related to the potential they move within

Higher enclosed mass  $\approx$  Higher stellar velocity

Measure stellar motions  $\rightarrow$  Infer mass distribution

But a complication!

We only measure velocity along line of sight:



Is orbit aligned with LOS and measured velocity is 3D velocity?

Or very misaligned and 3D velocity is ***much*** larger?



# Stellar dynamical modeling

Jeans modeling:

Assume:

Specific form for connection between  $v_{\text{LOS}}$  and  $v_{3\text{D}}$

Specific forms for galaxy shape (spherical or axisymmetric)



Schwarzschild orbit modeling:

Model full 3D orbit structure and find the  $v_{3\text{D}}$  distribution which is most consistent with  $v_{\text{LOS}}$ .

An extra quirk: the 3D galaxy shape impacts allowed orbits and allowed 3D orbit structure. Very general shapes  $\rightarrow$  very general orbit structures.

(The most general and flexible models are *triaxial Schwarzschild models*)

# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

Schwarzschild+79

Schwarzschild+93

# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

Schwarzschild+79

Schwarzschild+93

Propose a potential



# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

Schwarzschild+79

Schwarzschild+93

Propose a potential



Integrate  $\mathcal{O}(10^5)$  representative stellar orbits

# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

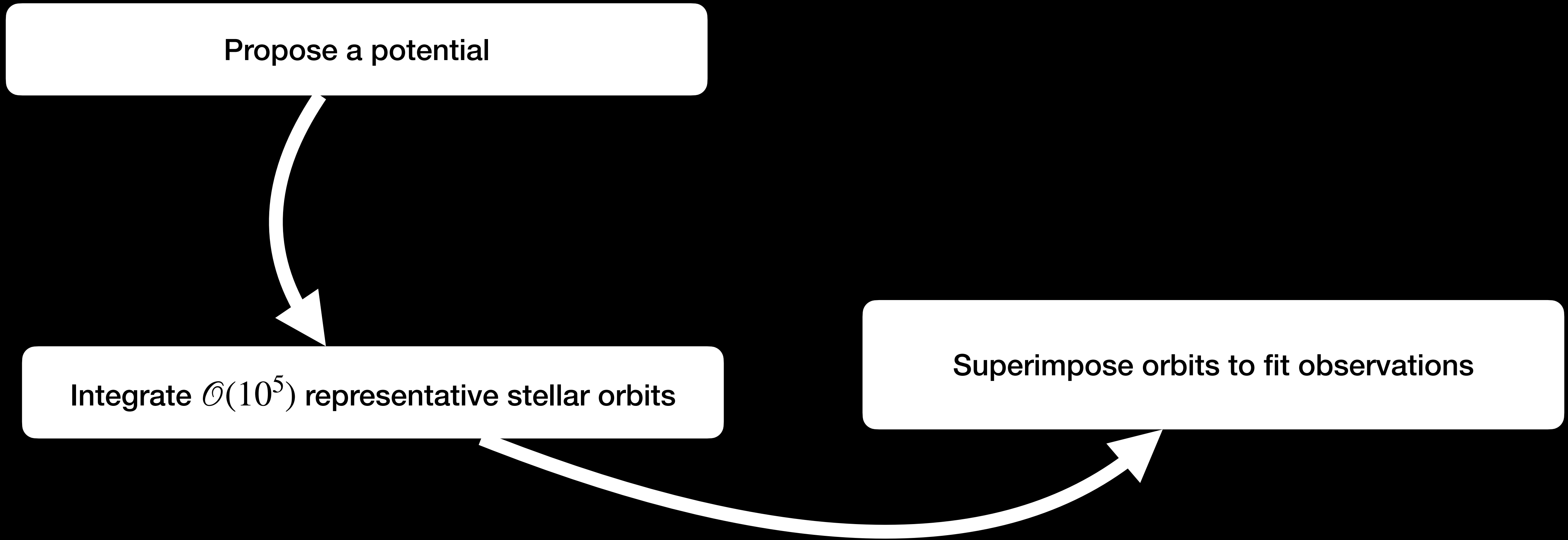
Schwarzschild+79

Schwarzschild+93

Propose a potential

Integrate  $\mathcal{O}(10^5)$  representative stellar orbits

Superimpose orbits to fit observations



# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

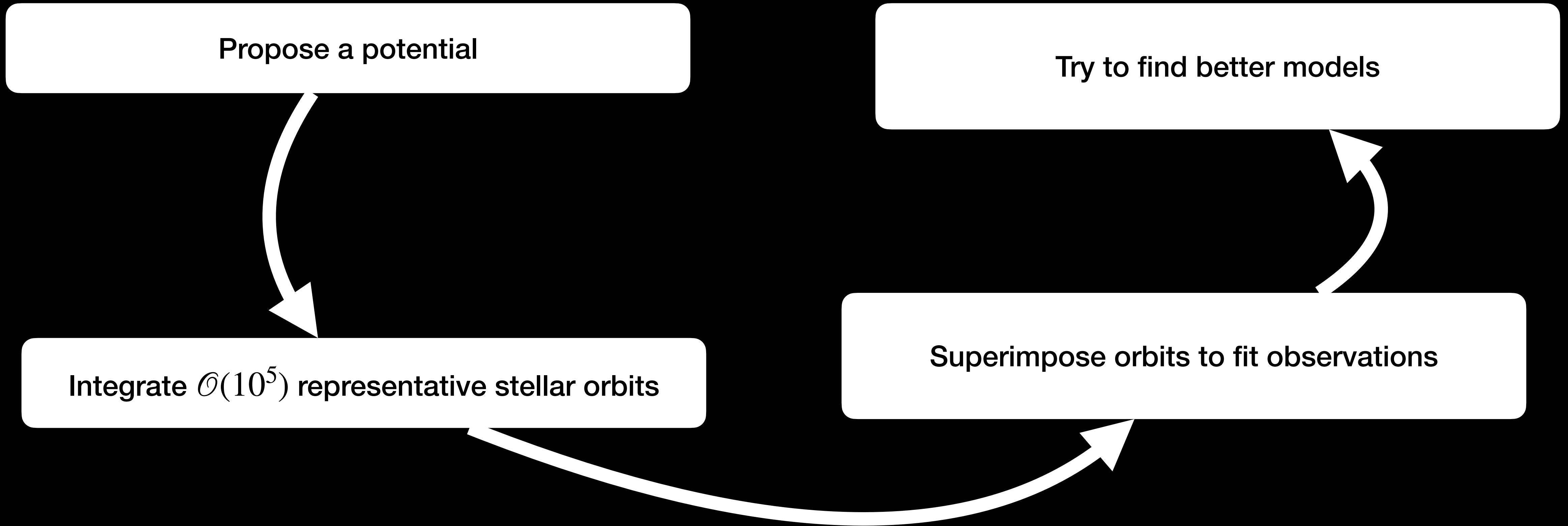
Schwarzschild+79  
Schwarzschild+93

Propose a potential

Try to find better models

Integrate  $\mathcal{O}(10^5)$  representative stellar orbits

Superimpose orbits to fit observations



# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

Schwarzschild+79  
Schwarzschild+93

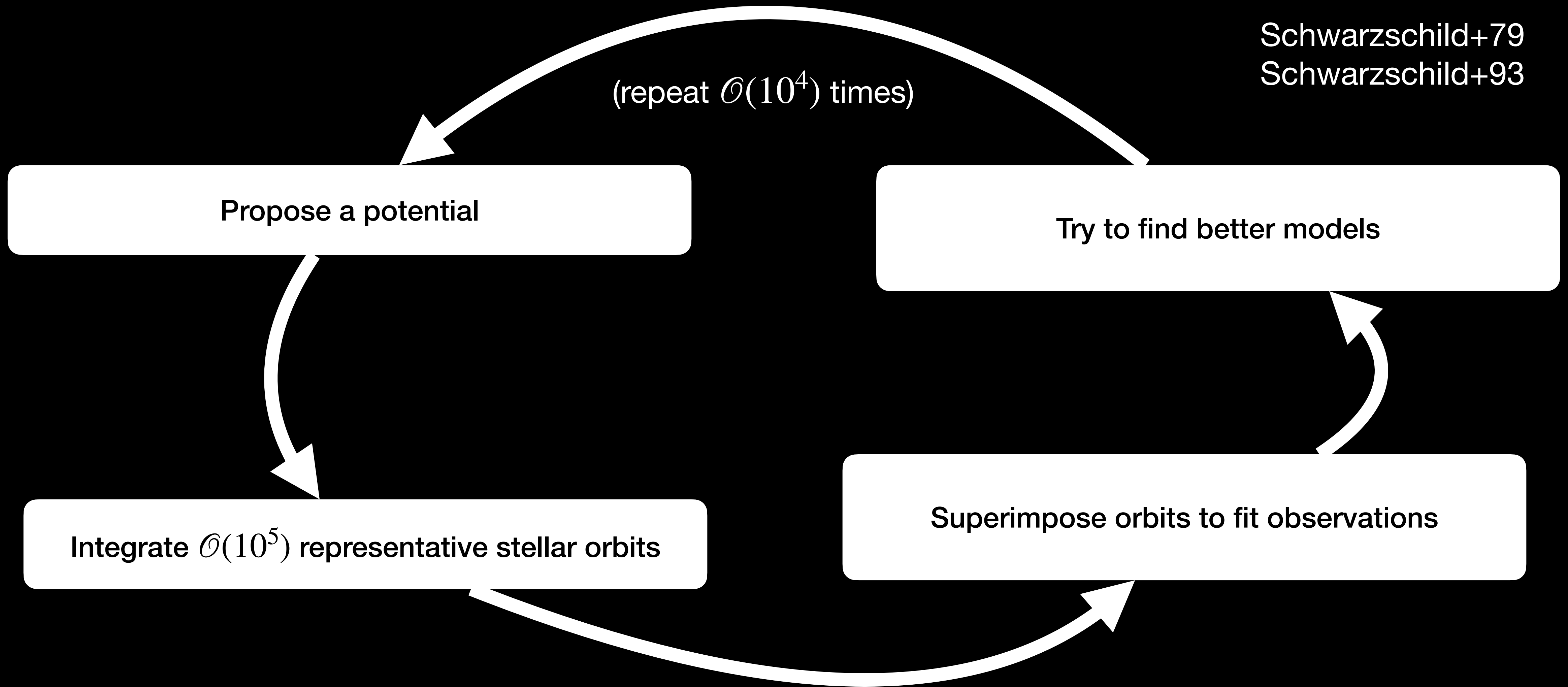
(repeat  $\mathcal{O}(10^4)$  times)

Propose a potential

Try to find better models

Integrate  $\mathcal{O}(10^5)$  representative stellar orbits

Superimpose orbits to fit observations



# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

Schwarzschild+79  
Schwarzschild+93

(repeat  $\mathcal{O}(10^4)$  times)

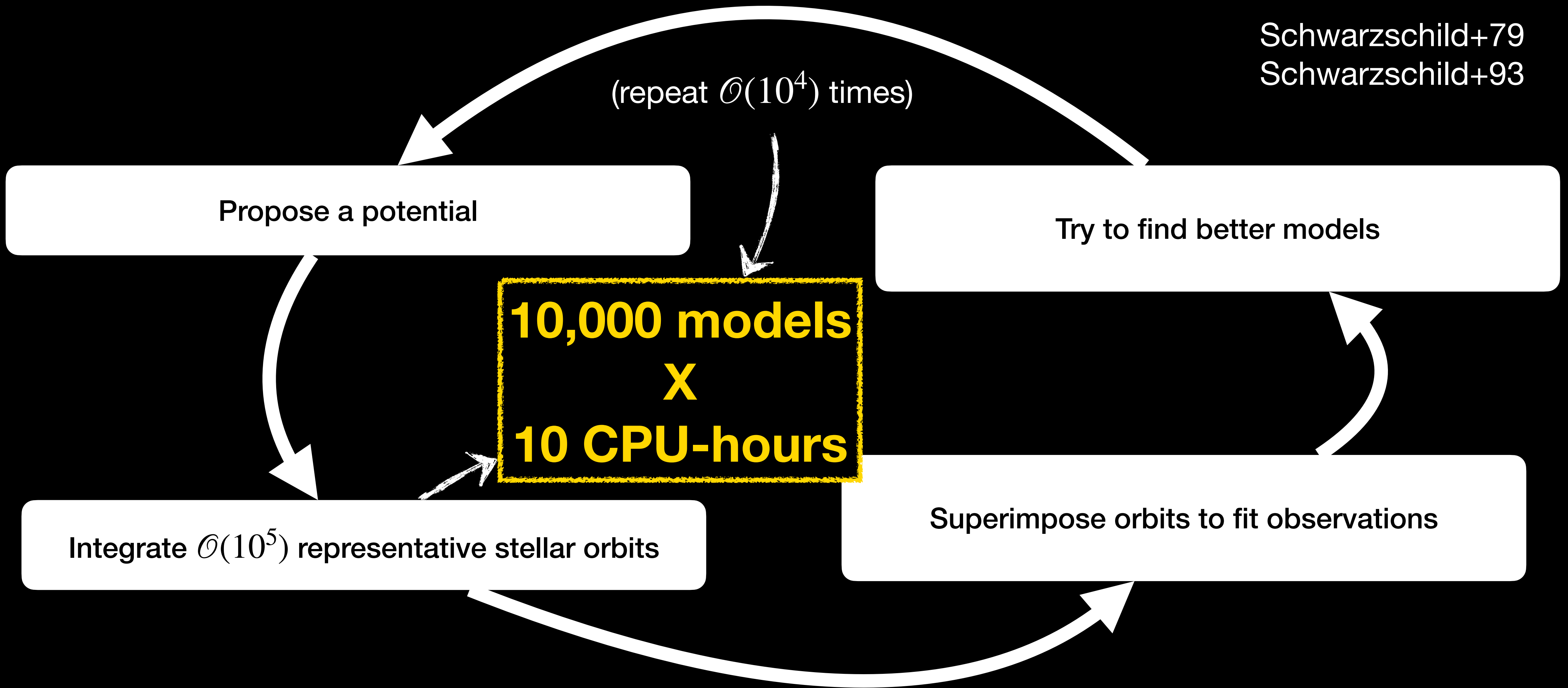
Propose a potential

Try to find better models

**10,000 models  
X  
10 CPU-hours**

Integrate  $\mathcal{O}(10^5)$  representative stellar orbits

Superimpose orbits to fit observations



# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

Schwarzschild+79  
Schwarzschild+93

(repeat  $\mathcal{O}(10^4)$  times)

Propose a potential

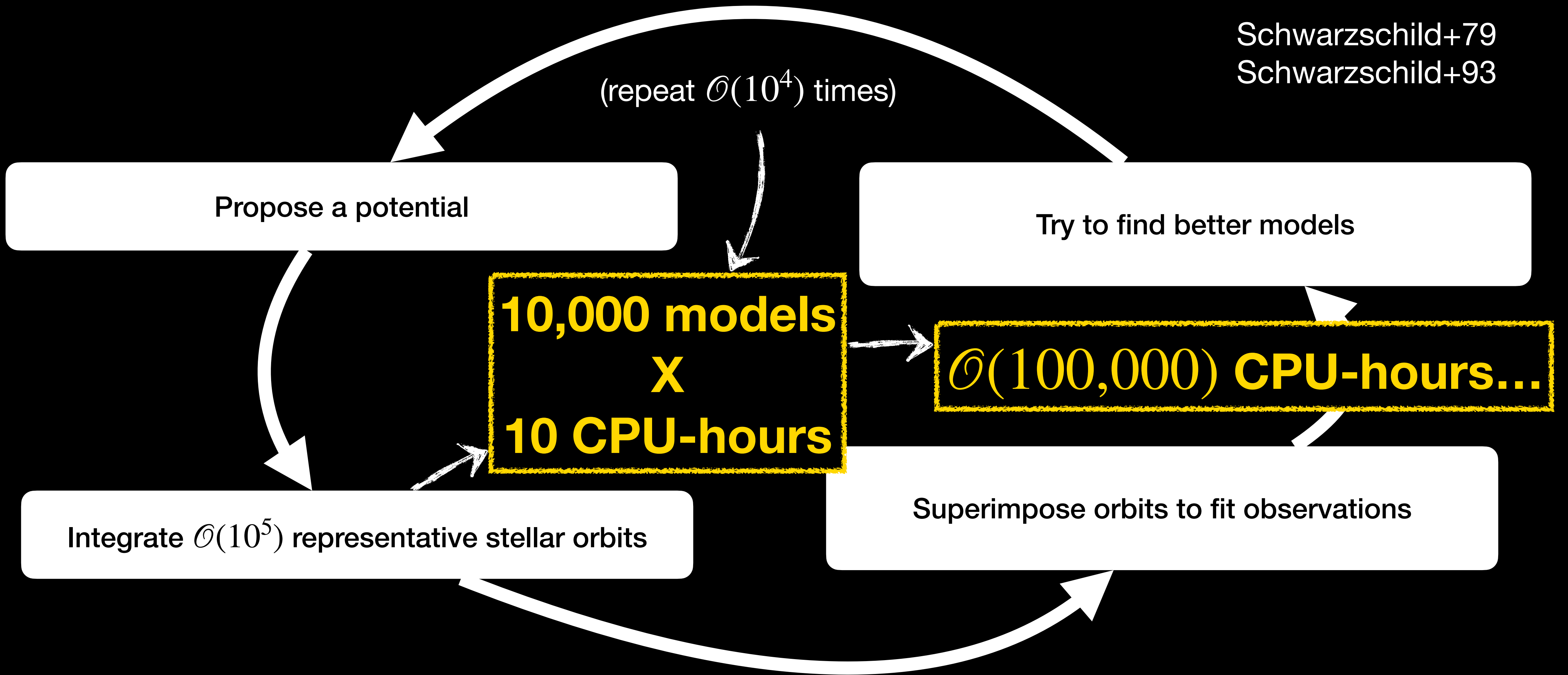
Try to find better models

**10,000 models  
X  
10 CPU-hours**

**$\mathcal{O}(100,000)$  CPU-hours...**

Integrate  $\mathcal{O}(10^5)$  representative stellar orbits

Superimpose orbits to fit observations





# How to **measure** SMBHs

## *Triaxial* Schwarzschild Modeling

Unfortunately, a large-ish parameter-space:

Black hole mass  
Stellar Mass-to-light ratio  
Dark matter halo mass (1 or 2 parameters)  
Intrinsic shape (3 parameters)

Schwarzschild+79

Schwarzschild+93

els

**X**

**10 CPU-hours**

**$\mathcal{O}(100,000)$  CPU-hours...**

Integrate  $\mathcal{O}(10^5)$  representative stellar orbits

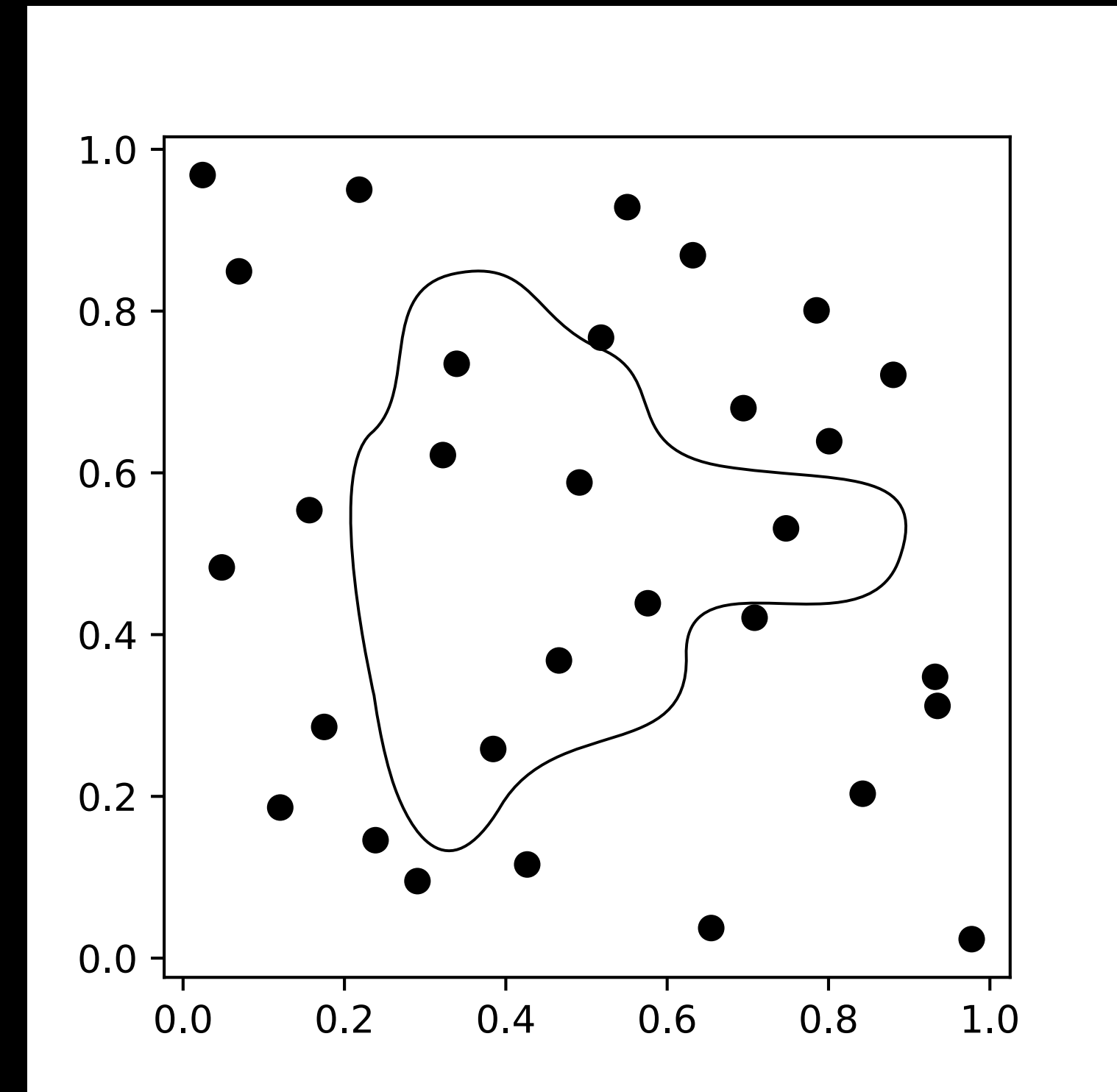
Superimpose orbits to fit observations

## Model Search using Gaussian Process regression and iterative search:

- Run uninformed set of models
- Model  $\chi^2$  surface with GP
- Populate low  $\Delta\chi^2$  volume
- Rinse and Repeat

## Dynamic Nested Sampling:

- Use GP Surrogate model
- Sample parameter space w/ Dynesty

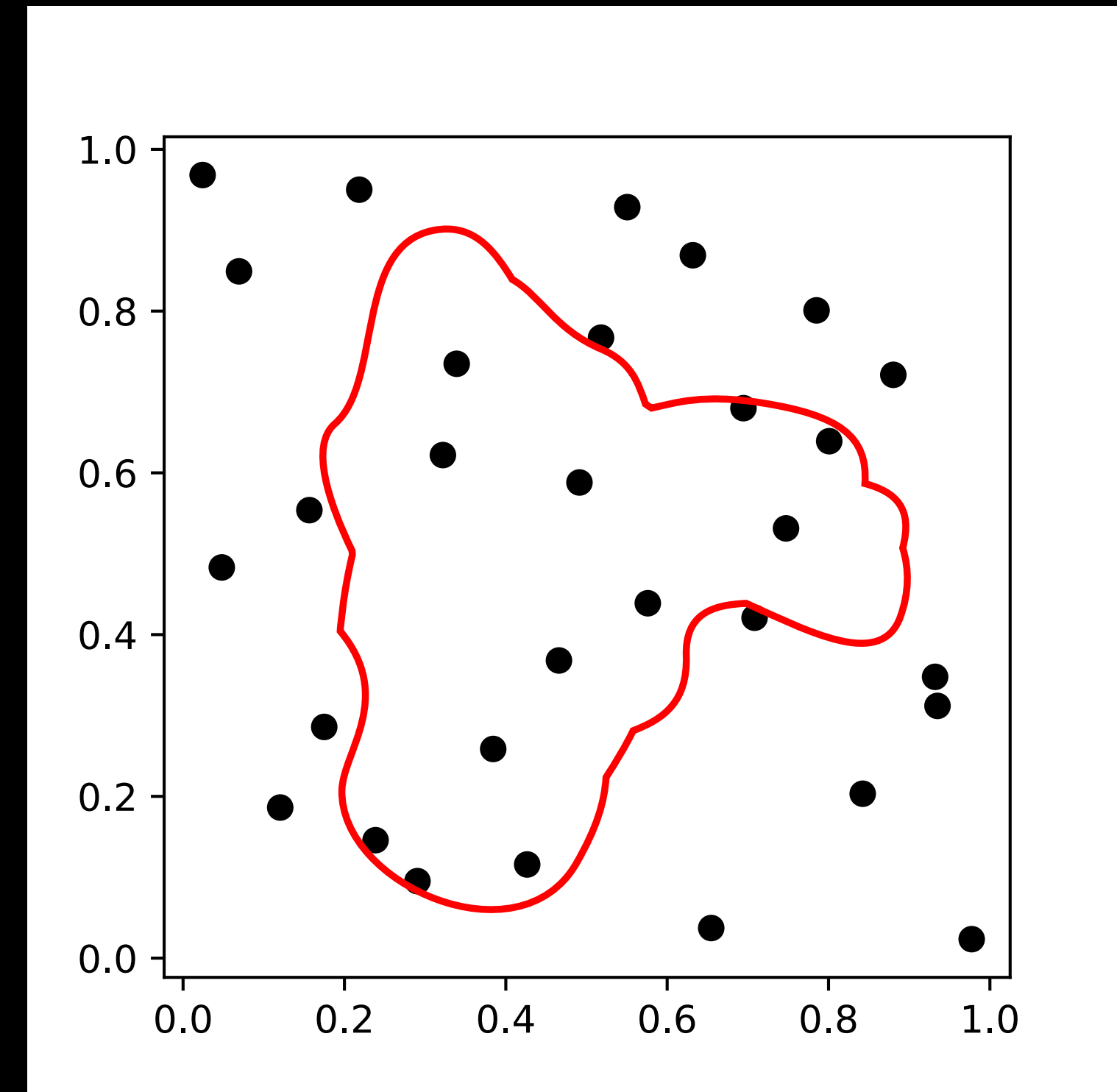


# Model Search using Gaussian Process regression and iterative search:

- Run uninformed set of models
- Model  $\chi^2$  surface with GP
- Populate low  $\Delta\chi^2$  volume
- Rinse and Repeat

## Dynamic Nested Sampling:

- Use GP Surrogate model
- Sample parameter space w/ Dynesty

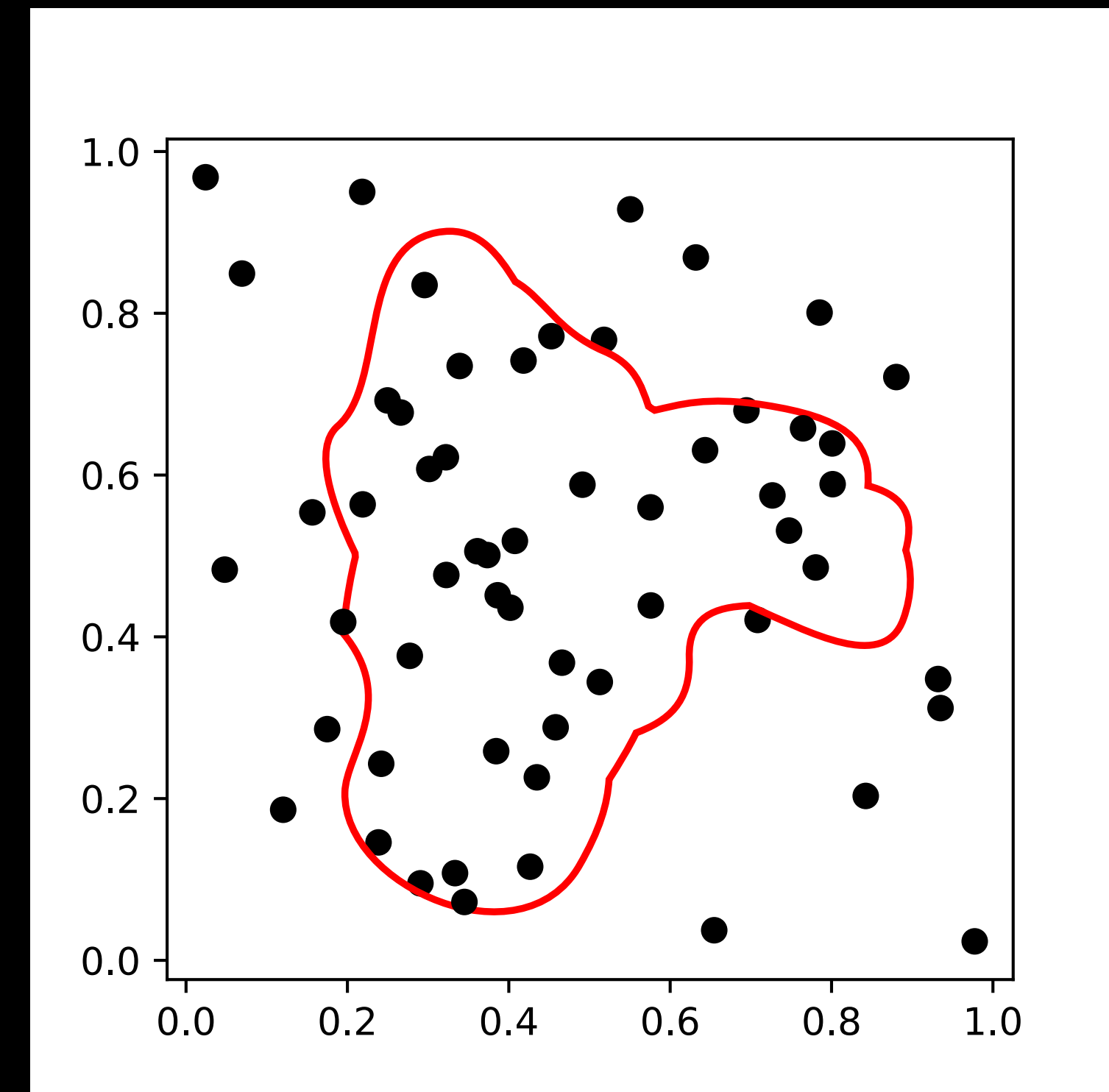


## Model Search using Gaussian Process regression and iterative search:

- Run uninformed set of models
- Model  $\chi^2$  surface with GP
- Populate low  $\Delta\chi^2$  volume
- Rinse and Repeat

## Dynamic Nested Sampling:

- Use GP Surrogate model
- Sample parameter space w/ Dynesty



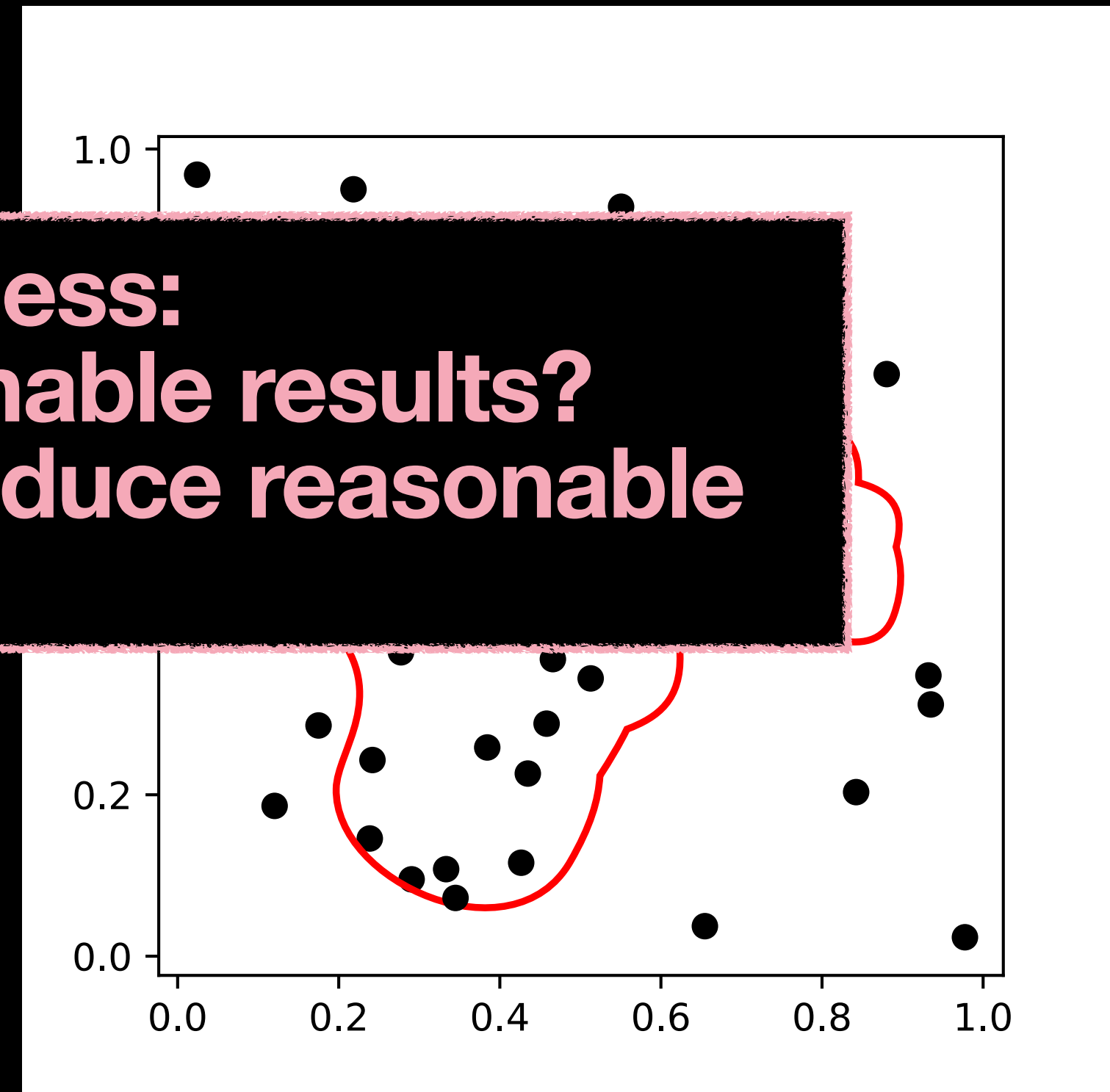
# Model Search using Gaussian Process regression and iterative search:

- Run uninformed set of models
- Model
- Pop
- Rinse and Repeat

## Dynamic Nested Sampling:

- Use GP Surrogate model
- Sample parameter space w/ Dynesty

Two layers of robustness:  
Do our models produce reasonable results?  
Does our parameter inference produce reasonable results?





# The robustness problem in stellar dynamical modeling

Given observations of a galaxy with a set of physical parameters, are our measurements both accurate and precise?

## Problem 1:

Triaxial orbit modeling is least assumption-laden stellar dynamical scheme.

Assumptions on galaxy shape, symmetry, orbital structure *may* be linked to biases.

→ We can't compare our models against more sophisticated / comprehensive models

## Problem 2:

No\* robust framework exists for constructing artificial galaxy data for *triaxial* galaxies with central SMBHs

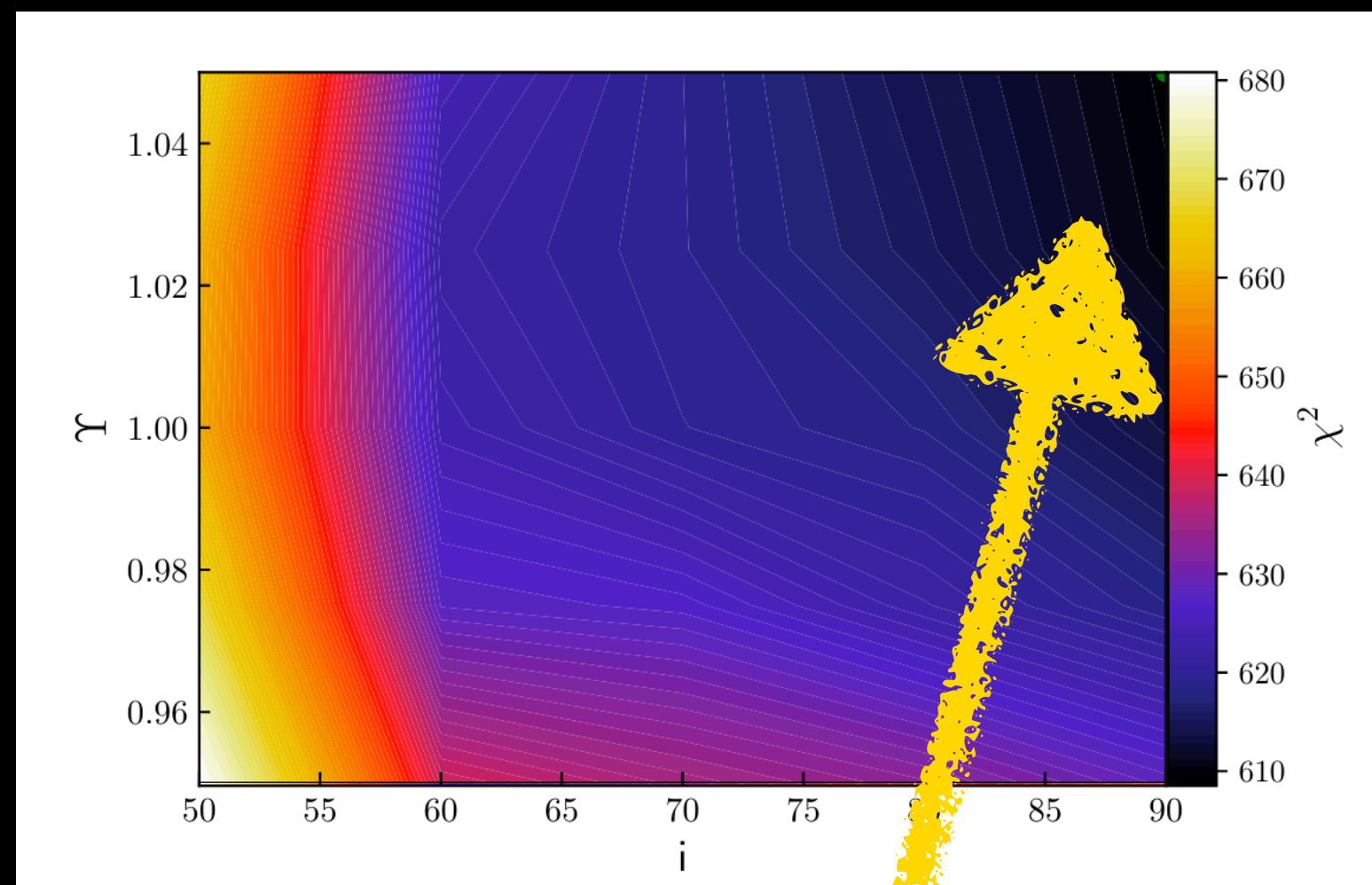
→ We can't\* test our models against reference models



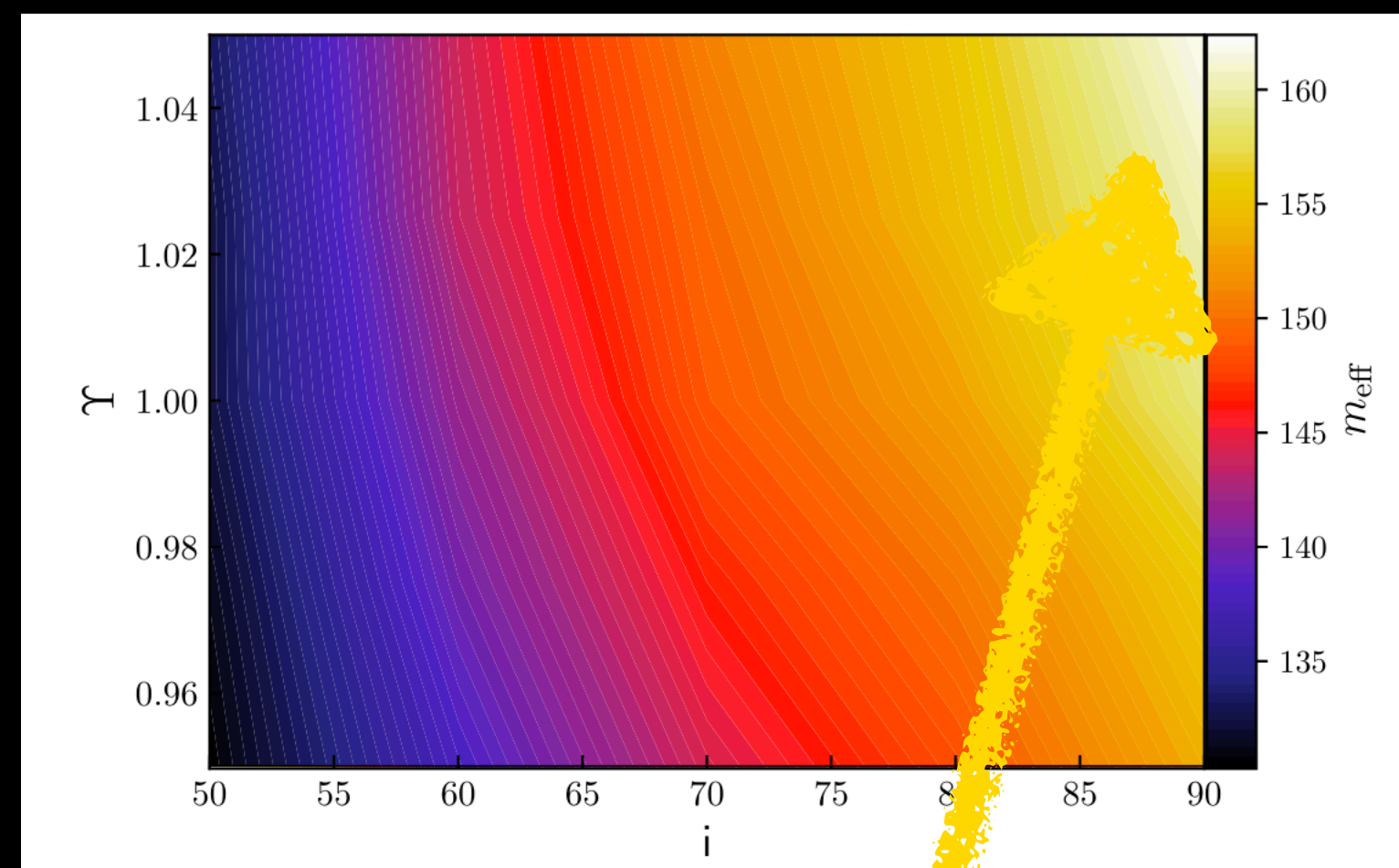
# Are orbit models inherently biased? (Lipka + Thomas 2021)

In nonlinear models, the number degrees of freedom *may* vary throughout the parameterspace

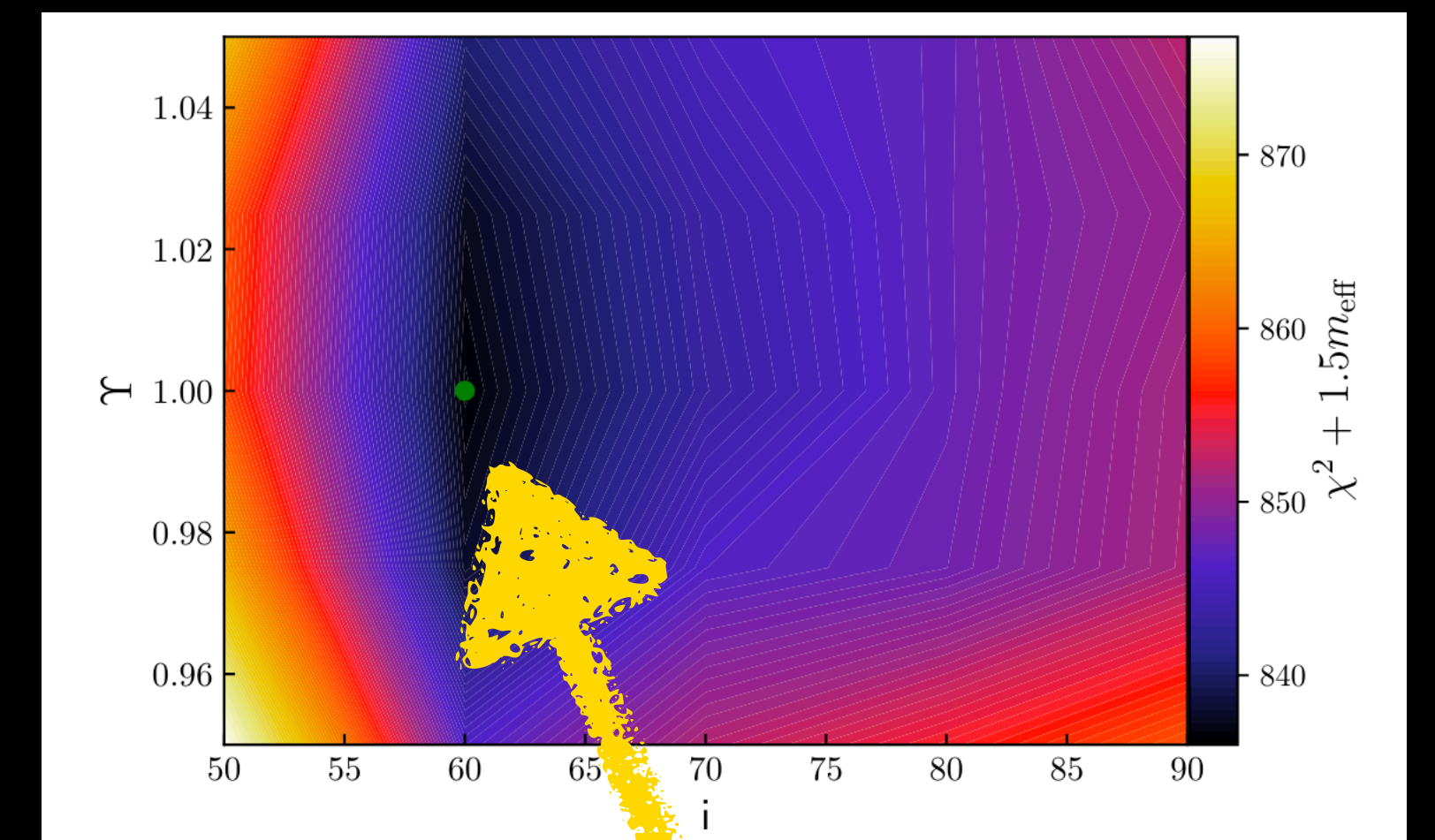
Lipka+Thomas explore number of generalized DOFs in axisymmetric orbit models



Goodness of fit



Number of DOF



Corrected  
Goodness of fit

# Probably not! (Pilawa, Liepold, Ma 2024)

Idea:

Use triaxial Schwarzschild models to create mock galaxy data from real galaxy data

1. Obtain real galaxy data
2. Fit real data with a triaxial orbit model with parameters  $\mathcal{P}_i$
3. Obtain predictions for observations from  $\mathcal{P}_i$  and nudge those predictions with random noise to produce synthetic observations  $\mathcal{O}_{i,j}$
4. Feed synthetic observations into modeling and inference
5. Repeat for many  $i$  and  $j$

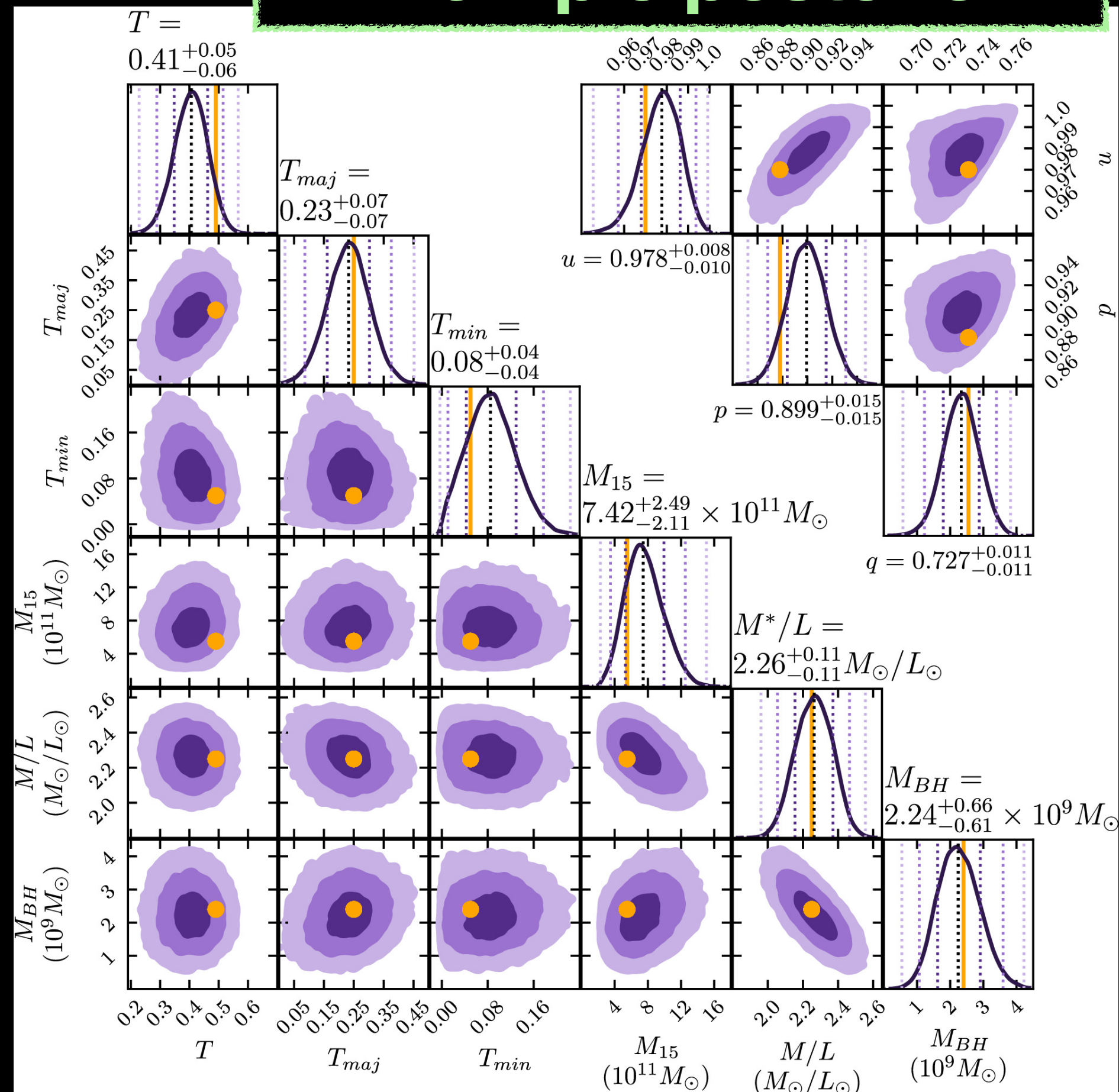
The resulting models are *equilibrium*, with *consistent kinematics* and *potential*



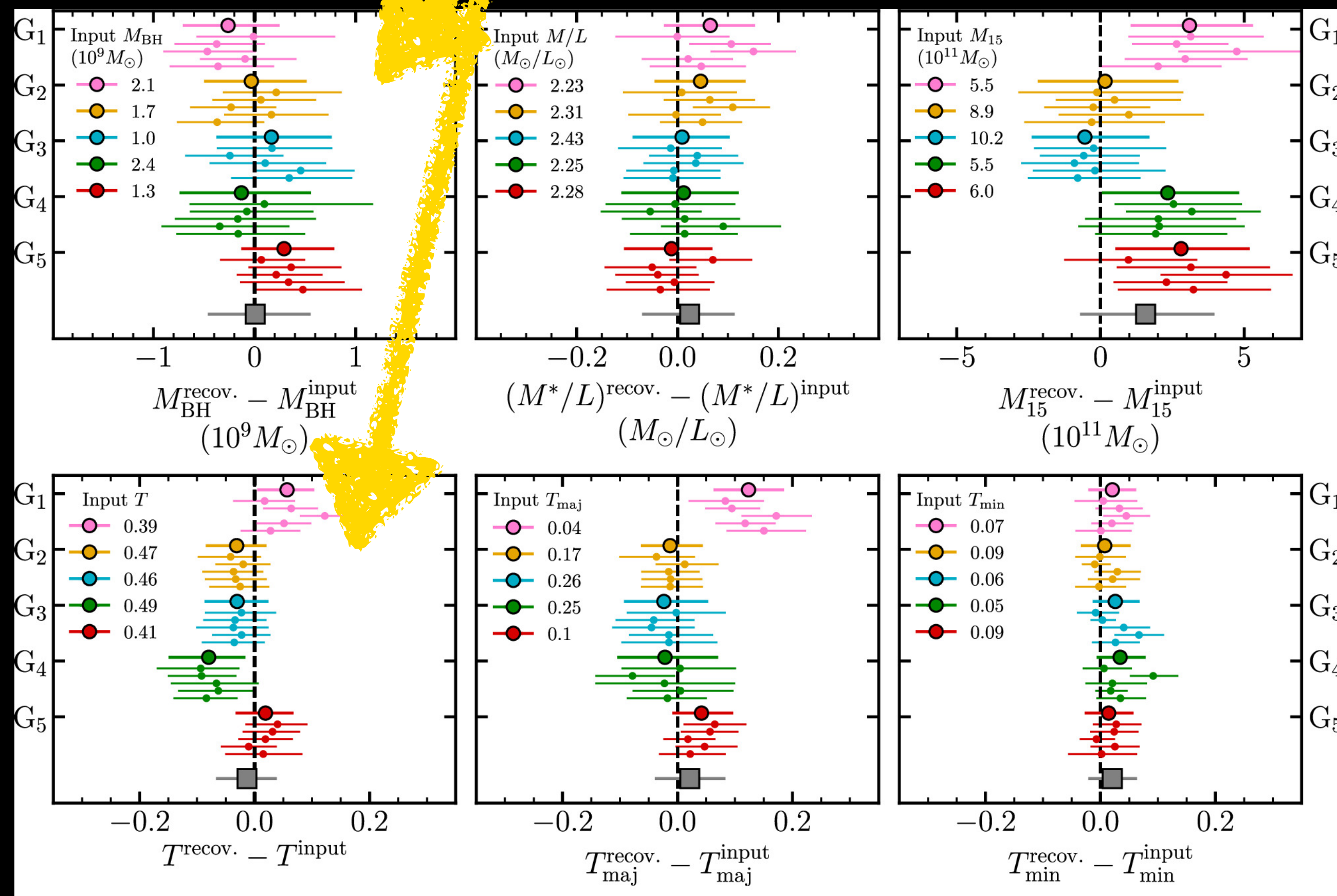
# Probably not! (Pilawa, Liepold, Ma 2024)

- Ran 5 models with 5 noise realizations apiece

## Example posterior



No apparent consistent bias in mass or shape parameters

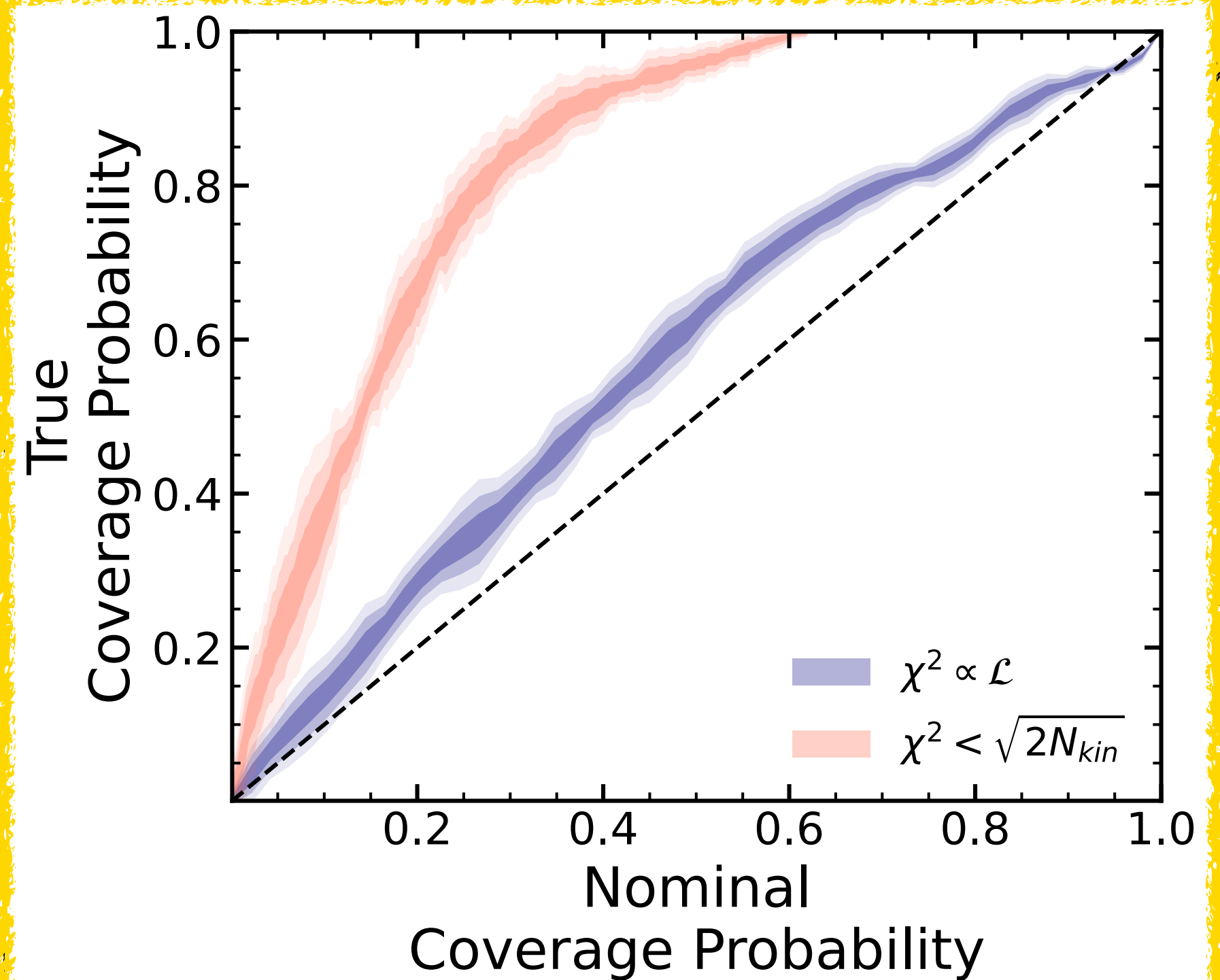
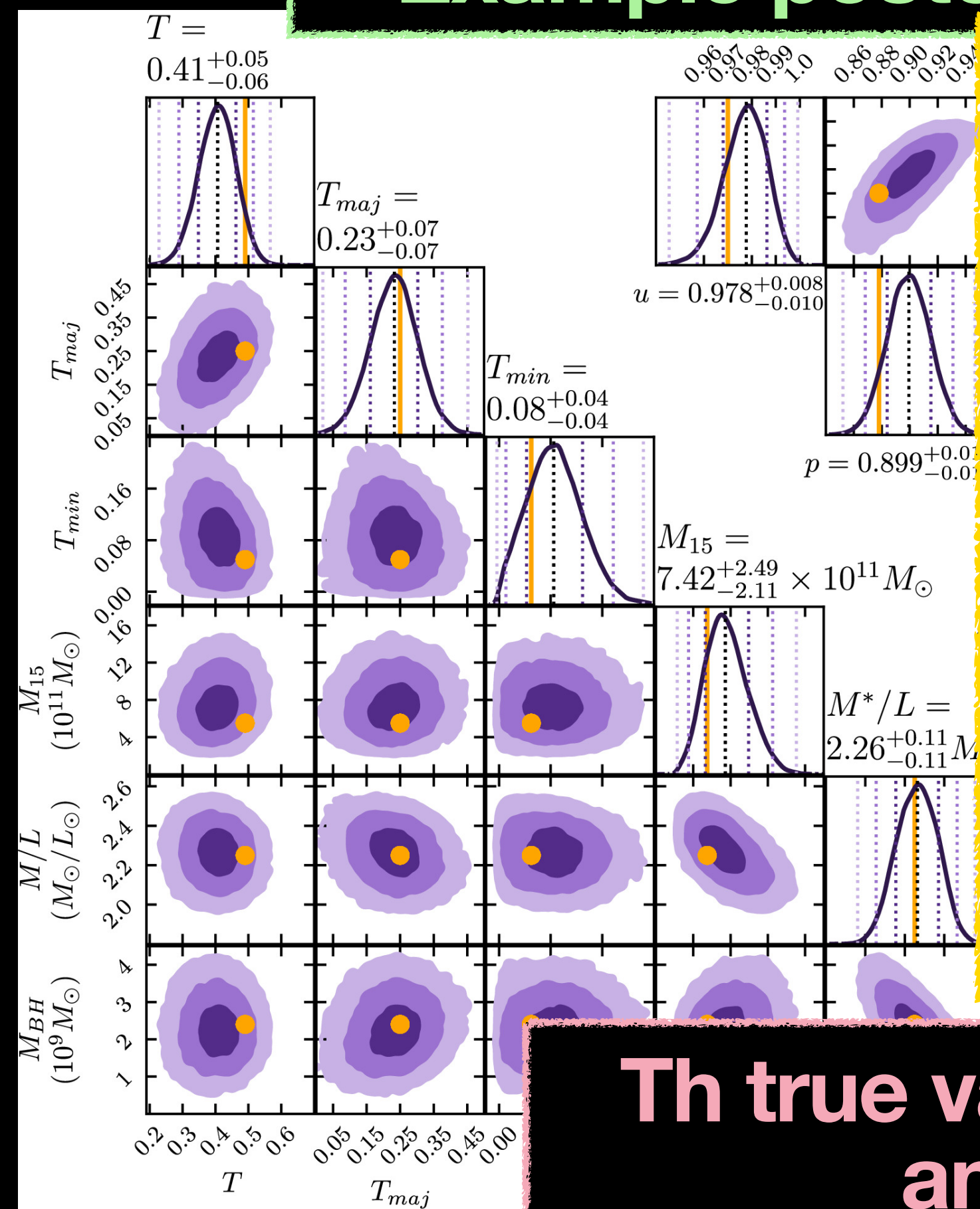




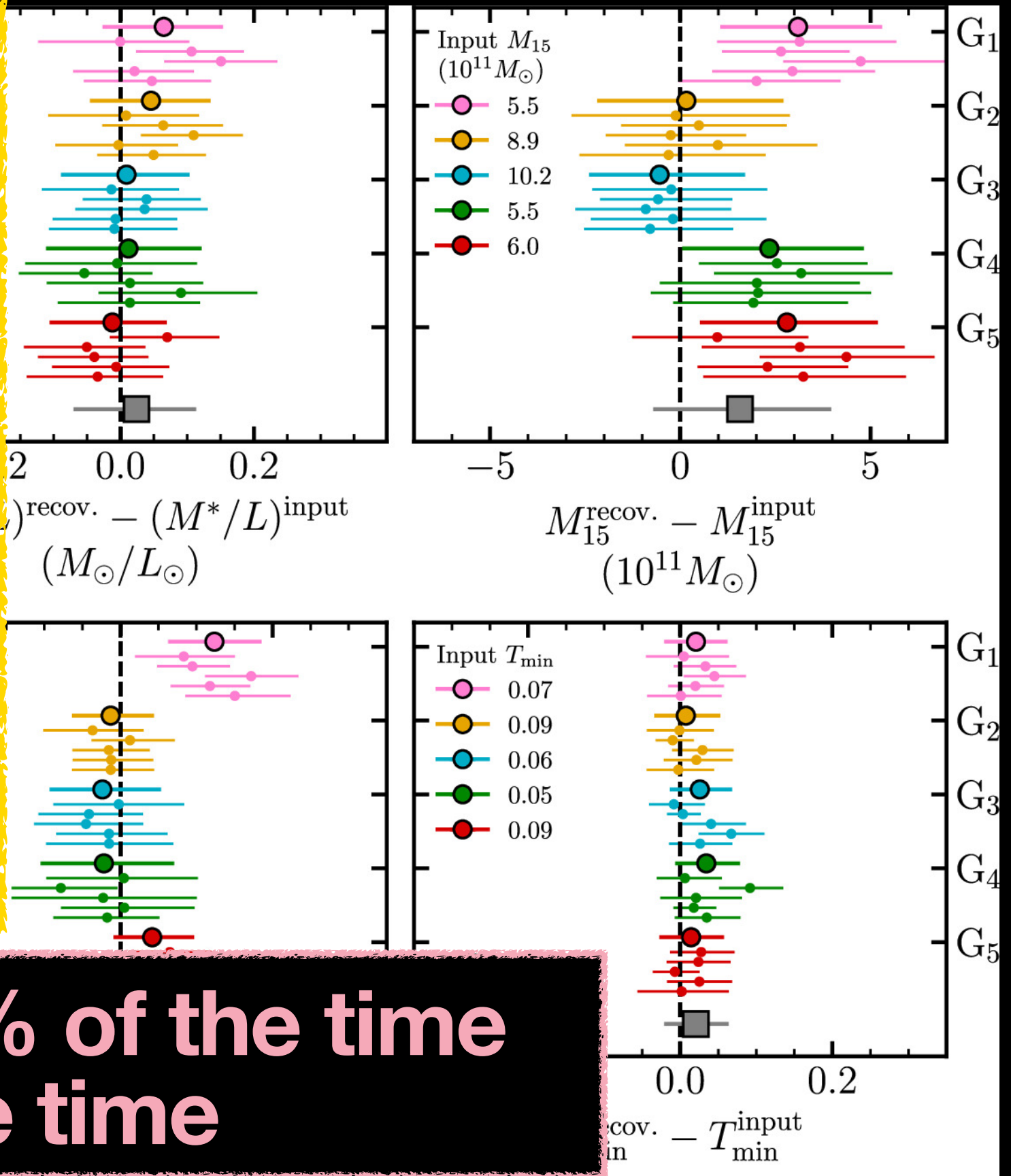
# Probably not! (Pilawa, Liepold, Ma 2024)

- Ran 5 models with 5 noise realizations apiece

## Example posterior



No apparent consistent bias in mass or shape parameters



The true value was within 68% CR ~75% of the time and within 95% CR ~95% of the time

**Black hole masses and galaxy properties are linked**

**Triaxial Orbit modeling is the most general stellar dynamical modeling method**

**But it's quite challenging to know if the answers are correct**

**(Though our tests suggest that they probably are)**

**And our iterative scheme seems to reliably converge on reasonable answers**