A Duet of Black Holes from the TriOS (Triaxial Orbit Superposition) Code

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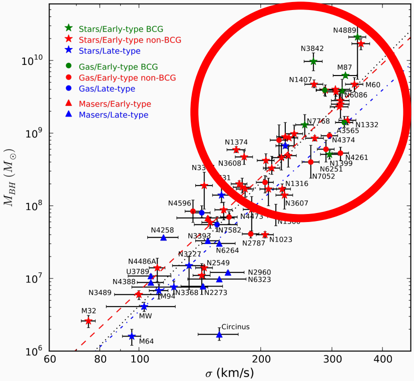
Motivation: What are we looking at?

The **MASSIVE** Survey targets **MASSIVE** galaxies with **MASSIVE** black holes



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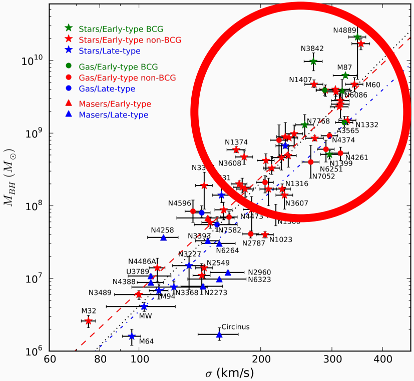
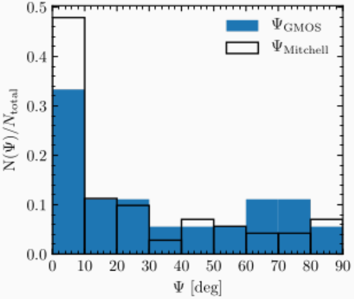


- These galaxies often have **kinematic misalignments**
- **Kinematic misalignments** strongly suggest a **triaxial** intrinsic shape (not axisymmetry!)

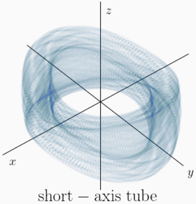
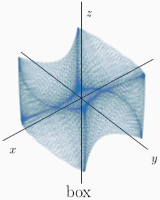


Motivation: Why do we care about the shape?

Shape of $\rho \rightarrow$ Shape of $\Phi \rightarrow$ *Symmetries* of $\Phi \rightarrow$ Conserved quantities and allowed orbits

Symmetry		Conserved Quantity	Orbits
Spherical	$\frac{d\Phi}{d\Omega} = 0$	$(E, \vec{L})$	Rosettes in fixed planes
Axisymmetry	$\frac{d\Phi}{d\phi} = 0$	(E, L_z, I_3)	Loops about symmetry axis
Triaxiality	Eh...	$(E, \mathbf{I}_2, \mathbf{I}_3)$	It's complicated...

Orbits in triaxial potentials

Loop Orbits	**Box** Orbits
	

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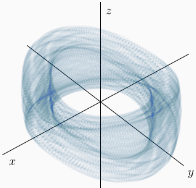	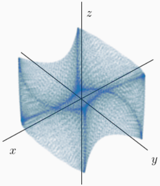
short − axis tube	box
Appears in axisymmetric potentials	Not present in axisymmetry!

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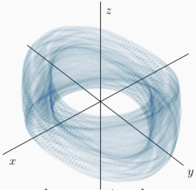
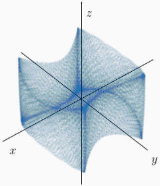
 short − axis tube	 box
Appears in axisymmetric potentials Persistent sense of rotation about either the **short** or **long** axis	Not present in axisymmetry! No persistent sense of rotation

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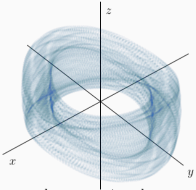	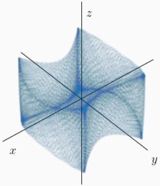
Appears in axisymmetric potentials Persistent sense of rotation about either the **short** or **long** axis **Centrophobic**	Not present in axisymmetry! No persistent sense of rotation Can be **Centrophilic**

Schwarzschild Orbit Modelling

Schwarzschild 1979: Can triaxial stellar systems in dynamical equilibrium be self-consistent?

Strategy:

1. Propose a (triaxial) stellar density distribution
2. Integrate representative orbits that span the phase space
3. Superimpose those orbits such that (1) is reproduced

The TriOS Triaxial Orbit Superposition Code

van den Bosch+ 2008: Development of a `fortan`-based code for Schwarzschild orbit modelling in triaxial stellar potentials.

Model includes BH, stars, and dark matter halo:

$$\Phi = \Phi_{BH} + \Phi_* + \Phi_{DM}$$

Stellar kinematics (LOSVDs) described by Gauss-Hermite expansion with $y = (v - V)/\sigma$:

$$f(v) = \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi\sigma^2}} \left[1 + \sum_{m=3}^{n} h_m H_m(y)\right]$$

2D (projected) and 3D (intrinsic) mass distributions are constrained for self-consistency.

The code was un-named. We call our improved version 'TriOS' (**Tri**axial **O**rbit **S**uperposition)

New Features in TriOS (Liepold+20) (Quenneville, Liepold, and Ma 2021a+b)

We've made a number of changes to the original **vdB+08** code

- We've added a mode which **axisymmetrizes** the orbits, effectively making TriOS an (optionally) axisymmetric code.
- Orbits in triaxial potentials were **improperly mirrored** in the original code. We've fixed this
- Orbits near the BH's sphere of influence precess ***slowly***. Orbits must be integrated for up to $\sim$2000 dynamical times before we find model convergence (vs 200 in previous usage)
- The original **orbit sampling density** leads to spurious biases in the preferred triaxiality parameter T. Doubling the sampling density solves the issue in most cases.
- Re-writing and tuning the routines for **PSF convolution** and **acceleration pre-caching** sped up the code by $5 - 10\times$ overall.
- In some circumstances, **energy conservation** was not checked after orbit integration or **mass self-consistency** was ill-enforced. These have been fixed.

Efficient Sampling for Triaxial Modelling (Liepold+ future paper)

Each `TriOS` model gives a χ^2 value for a single point in the parameter-space

- We need to search over M_{BH}, M/L (1 or 2 parameters), shape (3 parameters), and halo (1 or 2 parameters) – at least **6-8 dimensions**. (Grid Searches are inefficient)

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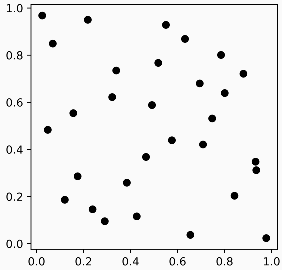
- This is **expensive**. Each model evaluation takes 10-30 CPU hours. (Highly iterative searches are impractical)
- As data improves, confidence volumes **shrink** with $\sim (\mathrm{Number\,of\,Constraints})^{-\mathrm{D}/2}$

Our Strategy (inspired by Bayesian Optimization and nested sampling):

1. Sparsely populate the space



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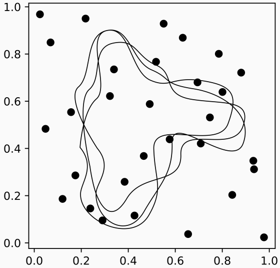

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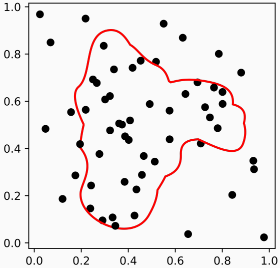

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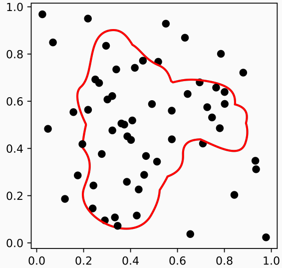
For our triaxial searches, we used this customized routine and only needed $3000 - 5000 \sim 4^6$ across 3 iterations for a $6D$ search. ($\sim$80,000 CPU-hours)



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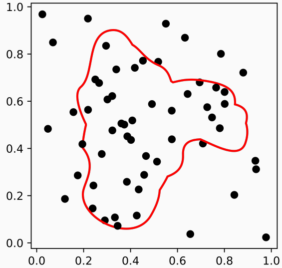
For a reasonable-resolution grid search (10 pt per dimension), we'd need $\mathrm{O}(10^6)$ models – 20,000,000 CPU-hours!



Efficient Sampling of the Shape (Quenneville, Liepold, and Ma 2021b)

- The 3D shape is determined through **deprojection** of the 2D surface brightness profile (we use MGEs)
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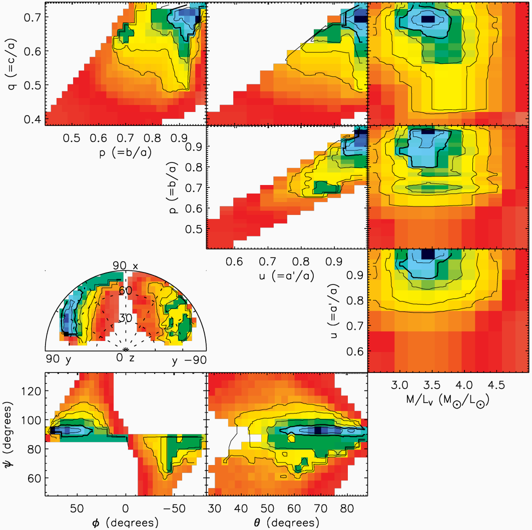

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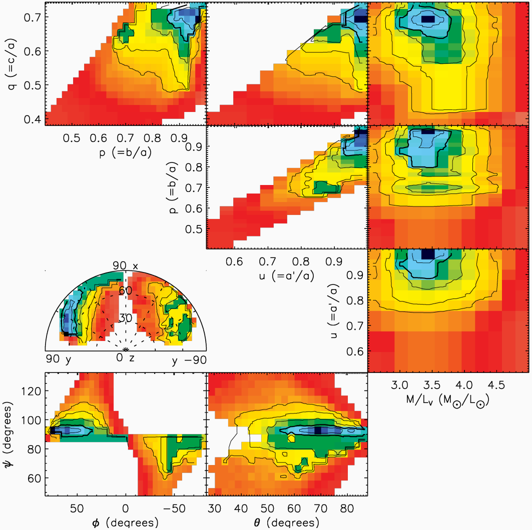
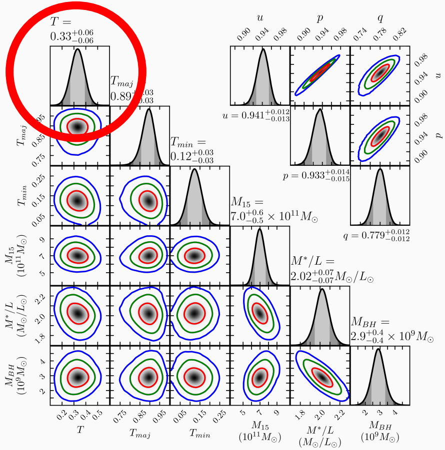
- Not all choices of these parameters produce valid deprojections ($0 \leq q \leq uq' \leq p \leq u \leq 1$)
- We've found an additional set of parameters which map the deprojectible shape space to a unit cube with minimal covariances

$$T = \frac{1-p^2}{1-q^2} \qquad T_{\mathrm{maj}} = \frac{1-u^2}{1-p^2} \qquad T_{\mathrm{min}} = \frac{(uq')^2-q^2}{p^2-q^2}$$



Triaxial NGC1453 and NGC2693 (Quenneville, Liepold, and Ma 2021b), (Pilawa, Liepold+22)

NGC1453



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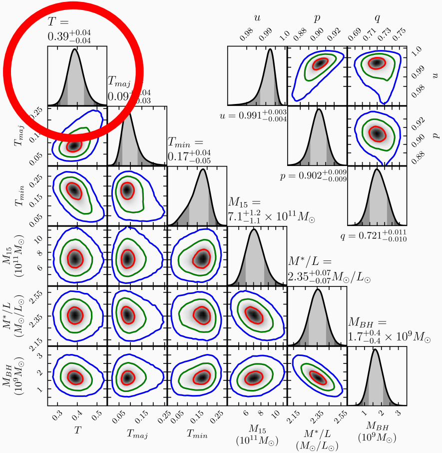

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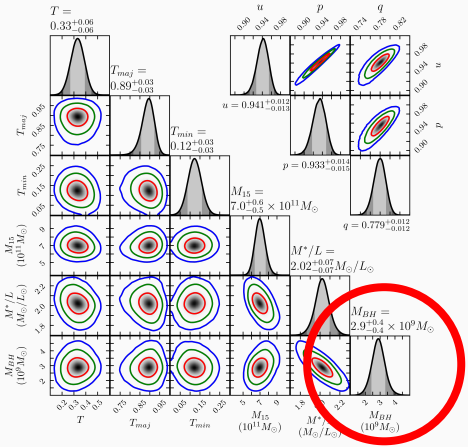

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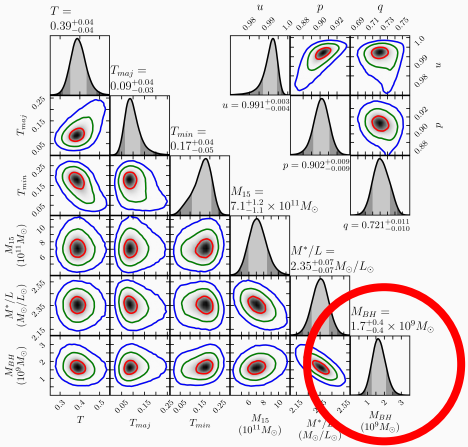

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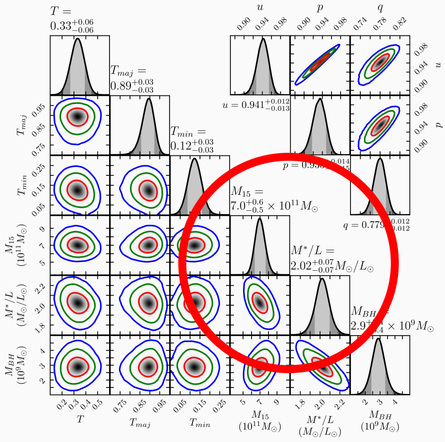

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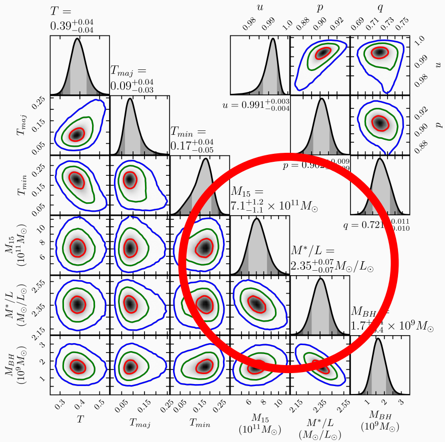

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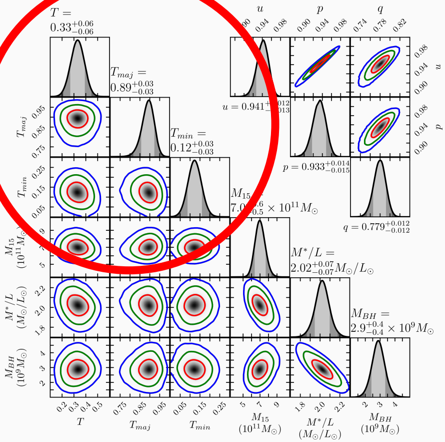

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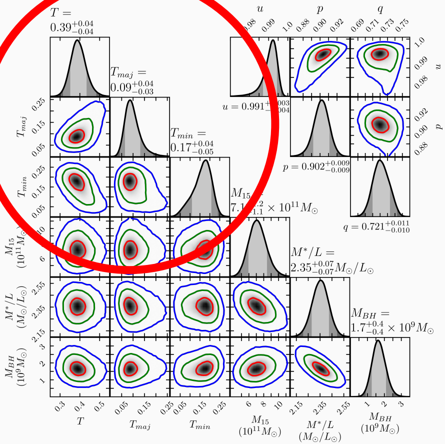
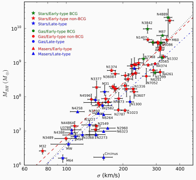


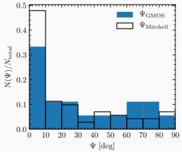
Looking Forward: Model Complexity

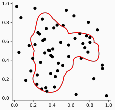
- Recent papers (Lipka+Thomas 2021+2022) suggest that the **model complexity** must be taken into account while finding the best-fit models by adding a penalty term to the model χ^2
- Our preliminary tests suggest that this issue is **significant for axisymmetric models** and a bias in inclination is present if the complexity is ignored
- Our preliminary tests find that the issue is **far less important for triaxial models** and the preferred shape is more-or-less unchanged when reasonable penalty terms are introduced.

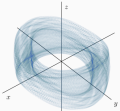
Thank you! (Questions?)

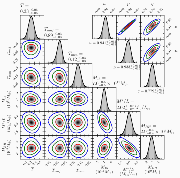
Looking Backward

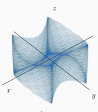


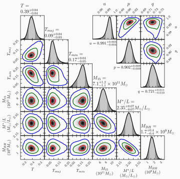












Looking Forward

- Improved search algorithms
- Model Complexity?
- **More Galaxies! (Exciting results on the horizon)**